

2^{nde} – 1^{ère} Corrigé de la feuille d'exercices sur les quantités conjuguées

$$A = \frac{1 + 2\sqrt{3}}{1 - 2\sqrt{3}}$$

$$A = \frac{(1 + 2\sqrt{3})^2}{(1 - 2\sqrt{3})(1 + 2\sqrt{3})}$$

$$A = \frac{1^2 + 2 \times 1 \times 2\sqrt{3} + (2\sqrt{3})^2}{1^2 - (2\sqrt{3})^2}$$

$$A = \frac{1 + 4\sqrt{3} + 4 \times 3}{1 - 4 \times 3}$$

$$A = \frac{13 + 4\sqrt{3}}{-11}$$

$$A = \boxed{\frac{-13 - 4\sqrt{3}}{11}}$$

$$B = \frac{2 + \sqrt{3}}{2 + \sqrt{7}}$$

$$B = \frac{(2 + \sqrt{3})(2 - \sqrt{7})}{(2 + \sqrt{7})(2 - \sqrt{7})}$$

$$B = \frac{4 + 2\sqrt{3} - 2\sqrt{7} - \sqrt{3}\sqrt{7}}{2^2 - (\sqrt{7})^2}$$

$$B = \frac{4 + 2\sqrt{3} - 2\sqrt{7} - \sqrt{21}}{4 - 7}$$

$$B = \frac{4 + 2\sqrt{3} - 2\sqrt{7} - \sqrt{21}}{-3}$$

$$B = \boxed{\frac{-4 - 2\sqrt{3} + 2\sqrt{7} + \sqrt{21}}{3}}$$

$$C = \frac{\sqrt{2} + 1}{\sqrt{2} + 2}$$

$$C = \frac{(\sqrt{2}+1)(\sqrt{2}-2)}{(\sqrt{2}+2)(\sqrt{2}-2)}$$

$$C = \frac{2 + \sqrt{2} - 2\sqrt{2} - 2}{(\sqrt{2})^2 - 2^2}$$

$$C = \frac{-\sqrt{2}}{2-4}$$

$$C = \frac{-\sqrt{2}}{-2}$$

$$C = \boxed{\frac{\sqrt{2}}{2}}$$

$$D = \frac{\sqrt{3} + 3}{\sqrt{3} + \sqrt{6}} = E$$

$$D = \frac{(\sqrt{3}+3)(\sqrt{3}-\sqrt{6})}{(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})}$$

$$D = \frac{3 + 3\sqrt{3} - \sqrt{3}\sqrt{6} - 3\sqrt{6}}{3 - 6}$$

$$D = \frac{3 + 3\sqrt{3} - 3\sqrt{6} - \sqrt{18}}{-3}$$

$$D = \frac{3 + 3\sqrt{3} - 3\sqrt{6} - 3\sqrt{2}}{-3}$$

$$D = \boxed{-1 + \sqrt{2} - \sqrt{3} + \sqrt{6}}$$

$$F = \frac{3\sqrt{3} + 2\sqrt{7}}{3\sqrt{3} - 2\sqrt{7}}$$

$$F = \frac{(3\sqrt{3} + 2\sqrt{7})^2}{(3\sqrt{3} - 2\sqrt{7})(3\sqrt{3} + 2\sqrt{7})}$$

$$F = \frac{(3\sqrt{3})^2 + 2 \times 3\sqrt{3} \times 2\sqrt{7} + (2\sqrt{7})^2}{(3\sqrt{3})^2 - (2\sqrt{7})^2}$$

$$F = \frac{9 \times 3 + 12\sqrt{3}\sqrt{7} + 4 \times 7}{9 \times 3 - 4 \times 7}$$

$$F = \frac{55 + 12\sqrt{21}}{-1}$$

$$F = \boxed{-55 - 12\sqrt{21}}$$

$$G = \sqrt{\frac{\sqrt{17} + 4}{\sqrt{17} - 4}}$$

$$G = \sqrt{\frac{(\sqrt{17} + 4)^2}{(\sqrt{17} - 4)(\sqrt{17} + 4)}}$$

$$G = \sqrt{\frac{(\sqrt{17} + 4)^2}{17 - 16}}$$

$$G = \sqrt{\frac{(\sqrt{17} + 4)^2}{1}}$$

$$G = \sqrt{(\sqrt{17} + 4)^2}$$

$$G = \boxed{\sqrt{17} + 4} \text{ car } \sqrt{17} + 4 > 0$$

$$H = \frac{2}{\sqrt{5} + 1} + \frac{5}{5 - \sqrt{5}}$$

$$H = \frac{2(\sqrt{5} - 1)}{(\sqrt{5} + 1)(\sqrt{5} - 1)} + \frac{5(5 + \sqrt{5})}{(5 - \sqrt{5})(5 + \sqrt{5})}$$

$$H = \frac{2\sqrt{5} - 2}{5 - 1} + \frac{25 + 5\sqrt{5}}{25 - 5}$$

$$H = \frac{2\sqrt{5} - 2}{4} + \frac{25 + 5\sqrt{5}}{20}$$

$$H = \frac{2\sqrt{5} - 2}{4} + \frac{5 + \sqrt{5}}{4}$$

$$H = \frac{2\sqrt{5} - 2 + 5 + \sqrt{5}}{4} = \boxed{\frac{3 + 3\sqrt{5}}{4}}$$

$$I = \frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$I = \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$I = \frac{(5\sqrt{5} + \sqrt{15})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} - \frac{\sqrt{15}-3}{5-3}$$

$$I = \frac{(5 + \sqrt{15}) - (\sqrt{15} - 3)}{5-3}$$

$$I = \frac{5 + \sqrt{15} - \sqrt{15} + 3}{2}$$

$$I = \frac{8}{2} \quad I = \boxed{4}$$

$$J = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$J = \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} + \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$J = \frac{3+2\sqrt{3}\sqrt{2}+2}{3-2} + \frac{3-2\sqrt{3}\sqrt{2}+2}{3-2}$$

$$J = \frac{5+2\sqrt{3}\sqrt{2}}{1} + \frac{5-2\sqrt{3}\sqrt{2}}{1}$$

$$J = \boxed{10}$$

$$K = \frac{3}{\sqrt{5}-\sqrt{2}} + \frac{4}{\sqrt{6}+\sqrt{2}}$$

$$K = \frac{3(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} + \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$

$$K = \frac{3\sqrt{5}+3\sqrt{2}}{5-2} + \frac{4\sqrt{6}-4\sqrt{2}}{6-2}$$

$$K = \frac{3\sqrt{5}+3\sqrt{2}}{3} + \frac{4\sqrt{6}-4\sqrt{2}}{4}$$

$$K = \sqrt{5} + \sqrt{2} + \sqrt{6} - \sqrt{2}$$

$$K = \boxed{\sqrt{5} + \sqrt{6}}$$

$$L = \frac{4}{3-\sqrt{5}}$$

$$L = \frac{4(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$L = \frac{12+4\sqrt{5}}{9-5}$$

$$L = \frac{12+4\sqrt{5}}{4}$$

$$L = \boxed{3+\sqrt{5}}$$

$$M = \frac{3-\sqrt{5}}{\sqrt{3}+\sqrt{5}}$$

$$M = \frac{(3-\sqrt{5})(\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})}$$

$$M = \frac{3\sqrt{3}-\sqrt{5}\sqrt{3}-3\sqrt{5}+5}{3-5}$$

$$M = \frac{5+3\sqrt{3}-3\sqrt{5}-\sqrt{15}}{-2}$$

$$M = \boxed{\frac{-5-3\sqrt{3}+3\sqrt{5}+\sqrt{15}}{2}}$$

$$N = \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{2}-1}$$

$$N = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$N = \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{2}+1}{2-1}$$

$$N = \sqrt{2} - 1 + \sqrt{2} + 1$$

$$N = \boxed{2\sqrt{2}}$$

$$O = \frac{2\sqrt{3}}{\sqrt{3}+3} + \frac{\sqrt{3}}{\sqrt{3}-3}$$

$$O = \frac{2\sqrt{3}(\sqrt{3}-3)}{(\sqrt{3}+3)(\sqrt{3}-3)} + \frac{\sqrt{3}(\sqrt{3}+3)}{(\sqrt{3}-3)(\sqrt{3}+3)}$$

$$O = \frac{6-6\sqrt{3}}{-6} + \frac{3+3\sqrt{3}}{-6} \quad O = \frac{9-3\sqrt{3}}{-6} \quad O = \boxed{\frac{-3+\sqrt{3}}{2}}$$

$$P = \frac{\sqrt{3}-1}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}}$$

$$P = \frac{(\sqrt{3}+1)(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$P = \frac{2\sqrt{3}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$P = \frac{2 \times 3 - 2\sqrt{3}\sqrt{2}}{3-2}$$

$$P = \frac{6-2\sqrt{6}}{1}$$

$$P = \boxed{6-2\sqrt{6}}$$

ou bien $P = \frac{(\sqrt{3}-1)(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} +$

$$P = \frac{3-\sqrt{3}-\sqrt{3}\sqrt{2}+\sqrt{2}}{3-2} + \frac{3+\sqrt{3}-\sqrt{3}\sqrt{2}-\sqrt{2}}{3-2}$$

$$P = \frac{6-2\sqrt{3}\sqrt{2}}{1}$$

$$P = \boxed{6-2\sqrt{6}}$$

$$Q = \frac{2}{\sqrt{6}-\sqrt{5}} - \frac{\sqrt{15}-\sqrt{3}}{\sqrt{6}-1}$$

$$Q = \frac{2(\sqrt{6}+\sqrt{5})}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})} - \frac{(\sqrt{15}-\sqrt{3})(\sqrt{6}+1)}{(\sqrt{6}-1)(\sqrt{6}+1)}$$

$$Q = \frac{2\sqrt{6}+2\sqrt{5}}{6-5} - \frac{\sqrt{15 \times 6} - \sqrt{3 \times 6} + \sqrt{15} - \sqrt{3}}{6-1}$$

$$Q = \frac{2\sqrt{6}+2\sqrt{5}}{1} - \frac{\sqrt{3 \times 3 \times 5 \times 2} - \sqrt{3 \times 3 \times 2} + \sqrt{15} - \sqrt{3}}{5}$$

$$Q = \frac{10\sqrt{6}+10\sqrt{5}}{5} - \frac{3\sqrt{10}-3\sqrt{2}+\sqrt{15}-\sqrt{3}}{5}$$

$$Q = \frac{(10\sqrt{6}+10\sqrt{5}) - (3\sqrt{10}-3\sqrt{2}+\sqrt{15}-\sqrt{3})}{5}$$

$$Q = \frac{10\sqrt{6}+10\sqrt{5}-3\sqrt{10}+3\sqrt{2}-\sqrt{15}+\sqrt{3}}{5}$$

$$Q = \boxed{\frac{3\sqrt{2}+\sqrt{3}+10\sqrt{5}+10\sqrt{6}-3\sqrt{10}-\sqrt{15}}{5}}$$

ou $Q = \boxed{\frac{3}{5}\sqrt{2} + \frac{\sqrt{3}}{5} + 2\sqrt{5} + 2\sqrt{6} - \frac{3}{5}\sqrt{10} - \frac{\sqrt{15}}{5}}$

$$R = \frac{7+2\sqrt{10}}{\sqrt{2}+\sqrt{5}} - \frac{7-2\sqrt{10}}{\sqrt{2}-\sqrt{5}}$$

$$R = \frac{-14\sqrt{5}+4\sqrt{20}}{-3}$$

$$R = \frac{(7+2\sqrt{10})(\sqrt{2}-\sqrt{5})}{(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5})} - \frac{(7-2\sqrt{10})(\sqrt{2}+\sqrt{5})}{(\sqrt{2}-\sqrt{5})(\sqrt{2}+\sqrt{5})}$$

$$R = \frac{7\sqrt{2}+2\sqrt{20}-7\sqrt{5}-2\sqrt{50}}{2-5} - \frac{7\sqrt{2}-2\sqrt{20}+7\sqrt{5}-2\sqrt{50}}{2-5}$$

$$R = \frac{-14\sqrt{5}+2\sqrt{20}-7\sqrt{5}-2\sqrt{50}-7\sqrt{2}+2\sqrt{20}-7\sqrt{5}+2\sqrt{50}}{-3}$$

$$R = \frac{-14\sqrt{5}+4\sqrt{4 \times 5}}{-3}$$

$$R =$$

$$R = \frac{-6\sqrt{5}}{-3} = \boxed{2\sqrt{5}}$$

$$\begin{aligned}
S &= \frac{5 - \sqrt{3}}{5 + \sqrt{3}} - \frac{5 + \sqrt{3}}{5 - \sqrt{3}} \\
S &= \frac{(5 - \sqrt{3})^2}{(5 + \sqrt{3})(5 - \sqrt{3})} - \frac{(5 + \sqrt{3})^2}{(5 - \sqrt{3})(5 + \sqrt{3})} \\
S &= \frac{25 - 10\sqrt{3} + 3}{25 - 3} - \frac{25 + 10\sqrt{3} + 3}{25 - 3} \\
S &= \frac{28 - 10\sqrt{3}}{22} - \frac{28 + 10\sqrt{3}}{22} \\
S &= \frac{(28 - 10\sqrt{3}) - (28 + 10\sqrt{3})}{22} \\
S &= \frac{28 - 10\sqrt{3} - 28 - 10\sqrt{3}}{22} \\
S &= \frac{-20\sqrt{3}}{22} \\
S &= \boxed{-\frac{10\sqrt{3}}{11}}
\end{aligned}$$